

SYBA

27/4/18

2 to 5 pm

pgs. 4

Statistics - II

Q.P. Code: 36245

25

(3 Hours)

Total Marks: 100

- N.B.**
1. All questions are compulsory.
 2. Attempt any two sub questions out of three from Q2, Q3, Q4 and Q5.
 3. Figures to the right indicate full marks.
 4. Use of non-programmable calculator is allowed.
 5. Standard normal table values are given at the end of the question paper.

Q.1 Attempt ANY FIVE sub-questions: (20)

a. (10)

i. If a continuous random variable x has probability density function,

$$f(x) = \frac{1}{3} \quad -1 \leq x \leq 0$$

$$= \frac{2}{3} \quad 0 \leq x \leq 1, \text{ then } E(x) \text{ is equal to } \dots$$

ii.

If $f(x)$ is a p.d.f then $\int_{-\infty}^{\infty} f(x) = 0$, true or false? Give reason.

iii.

If X follows exponential distribution then the relationship between its mean and variance is

iv.

A random variable has continuous Rectangular distribution over the interval $[-1,3]$. This distribution has variance equal to

v.

For a normal distribution the 1st and the 3rd quartiles are 54.52 and 78.86, then the median is?

vi.

A function for estimating a parameter is called

vii.

The formula for standard error of the difference between proportions under the hypothesis $H_0: P_1 = P_2$ against $P_1 \neq P_2$ with usual notations is

b. Answer the following in one/ two sentences (any five) (10)

i. State any two properties of cumulative distribution function.

ii. If $f(x) = C$ for $-6 < x < 6$, is a p.d.f of random variable X then find C .

iii. If X follows Rectangular distribution $(4,12)$, then find its mean.

iv. If X follows Exponential distribution with parameter 2, then its median is

v. For a normal distribution, standard deviation of $x = 2$, then mean deviation is....

vi. Distinguish between one sided and two sided tests.

vii. Government agency claims that only 20% families are below poverty line. Whereas a citizens body claims that this percentage is much higher. Give suitable null and alternative hypothesis.

- Q.2 Attempt any TWO sub-questions: (20)**
- a. i. Define Probability density function of a continuous random variable X and state its properties. (04)
- ii. For a continuous random variable X with p.d.f (06)
- $$f(x) = 3(1-x)^2 \quad 0 < x \leq 1$$
- $$= 0, \quad \text{Otherwise}$$
- Find first quartile and third quartile.
- b. i. State the formulae for skewness and Kurtosis in terms of moments (03)
- for a continuous random variable X
- ii. The probability density function of a random variable X is given as (07)
- follows:
- $$f(x) = 6x(1-x) \quad 0 \leq X \leq 1$$
- $$= 0 \quad \text{Otherwise}$$
- Find variance of X and Median of X.
- c. The following is the p.d.f of a continuous random variable X. (10)
- $$f(x) = 2x \quad 0 < x < \frac{1}{2}$$
- $$= \frac{6(3-x)}{25} \quad \frac{1}{2} \leq x \leq 3$$
- $$= 0, \quad \text{otherwise}$$
- Find cumulative distribution function of X , $P(X < 0.2)$ and $P(X > 1)$
- Q.3 Attempt any TWO sub-questions: (20)**
- a. Define rectangular distribution and derive its mean and variance. (10)
- b. i. Define exponential distribution with parameter λ . Hence derive its (06)
- median.
- ii. If X follows Exponential distribution with parameter $\lambda = 5$. Find (04)
- $P(1 < X < 2)$ and $P(X \geq 1.5)$
- c. i. State the probability density function of a normally distributed (04)
- random variable. Also state its mean deviation, quartile deviation.
- 1 ii. Sack of grains packed by an automatic machine loader has an (06)
- average weight of 100 kg and standard deviation 2.5 kg. Assume the distribution of weight of sack to be normal ,
- I. Find the probability of getting a sack with weight over 96 kg but below 105 kg
- II. If the dealer rejects the sack with weight below 98.5 kg, how many sacks do you expect to be rejected in a lot of 3000 sacks?
- Q.4 Attempt any TWO sub-questions: (20)**
- a. i. Derive $100(1-\alpha)\%$ for a population proportion P. (05)
- ii. A company fleet which operates and manages car rentals for company employees found that the tire life time for their vehicles has a mean of 50000 miles and standard deviation of 3500 miles, What is the probability that the sample mean life time for these 50 vehicles exceeds 52000 miles.

- b. Define the following terms as applied to testing of hypothesis (10)
- Null hypothesis
 - Alternative Hypothesis
 - Critical region
 - Power of the test
 - Level of significance

- c. i. A coin is known to be biased such that $P(\text{Head}) = \frac{1}{3}$ or $\frac{2}{3}$. To test $H_0: P(H) = 1/3$ against $H_1: P(H) = 2/3$. It is decided to toss the coin 5 times and reject H_0 iff at least 3 heads are observed. Determine the probabilities of both the types of errors. (06)
- ii. A batch of 100 resistors has an average of 102 Ohms. Assuming the population standard deviation of 8 Ohms, test whether the population mean is 100 Ohms at significance level of 5%. (04)

Q.5

Attempt any TWO sub-questions:

- a. i. If the c.d.f of a random variable X is given by (20)
- $$F(x) = \begin{cases} 0 & x < 0 \\ \frac{6}{5} \left(\frac{x^2}{2} + \frac{x^3}{3} \right) & 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (03)$$

Find the p.d.f of X. Also draw the p.d.f taking any three suitable values of x.

- ii. For a continuous random variable X in usual notations prove that $V(aX + b) = a^2 V(X)$. (07)
- Also state the formula for $E(aX + b)$. Hence find $E(6X)$ when $E(x) = 6$
- b. i. Derive mean and Variance for a continuous random variable X following Exponential distribution. (10)
- c. i. What is the problem of testing of hypothesis? Explain the process of testing? (04)
- ii. According to 2010 Census, 11.4% of all housing units in the United States were vacant. A county supervisor wonders if her county is different from this. She randomly selects 850 housing units in her county and finds that 129 of the housing units are vacant. Test the hypothesis. (06)

Standard normal table

Q.P.Code: 36245

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

Note for example area to left of 2.27 is found by reading the number in the row 2.2 and under column 7 as 0.9884